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# The Spectrum of the Geomagnetic Activity Index Ap

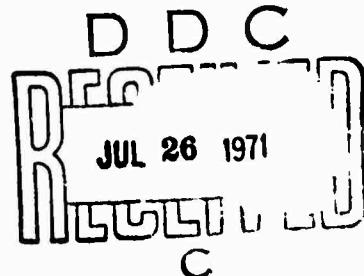
by

A. C. Fraser-Smith

May 1971

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12. ABSTRACT

The ionospheric propagation of high-frequency radio waves is profoundly influenced by the variations of sunspot numbers and of geomagnetic activity. Thus, details of these variations, and of the relationship between them, has practical application. The variation of the geomagnetic activity index Ap over the 38-year interval January, 1932 through January, 1970 differs in several important characteristics from the variation of sunspot numbers over the same interval. A spectral analysis approach was used to investigate this variation of Ap, as well as its relation to the variation of sunspot numbers. The data analyzed consisted of the 457 monthly average Ap indices (Ap) for the interval and the corresponding observed monthly sunspot numbers. Several large samples of the original daily Ap indices were also analyzed. Two of the latter samples covered 11-year intervals (April 1, 1944 - June 2, 1955; and June 1, 1956 - July 31, 1967) and the third covered a 23-year interval (April 1, 1944 - December, 1967).

The Ap spectrum is characterized by seven distinct lines with periods, in order of their relative line amplitudes, of 35.2 and 10.2 years, 6.0 months, 16.1, 7.0, 4.1 and 5.1 years. The lines with periods of 16.1, 7.0 and 5.1 years are not considered to be fundamental; they appear to be either harmonics or side-frequencies of the two strongest lines. The 10.2-year line in the Ap spectrum is matched by a sunspot line at exactly the same period. However, the sunspot cycle precedes the Ap cycle by 18 months. The semiannual variation has maxima close to the equinoxes. The error in the dates of the maxima is of the order of a few days, so the results clearly support the quinoctial hypothesis for the origin of the semiannual variation. The spectrum of the daily Ap indices is not dominated by the 27-day geomagnetic storm recurrence period,

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## Abstract (cont)

presumably due to the variation of this period over a solar cycle. Apart from an expected solar cycle component and its associated lines, the eight largest lines in the spectrum for the 23-year interval of Ap data occur at periods of 6.0 months, 1.47 years, 27.2 and 27.6 days, 1.23 months, 1.09 years, 13.7 and 14.1 days. There is no evidence for either an annual variation or a variation at the lunar synodic period.

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## ABSTRACT

The ionospheric propagation of high-frequency radio waves is profoundly influenced by the variations of sunspot numbers and of geomagnetic activity. Thus, details of these variations, and of the relationship between them, has practical application. The variation of the geomagnetic activity index  $\bar{A}_p$  over the 38-year interval January, 1932 through January, 1970 differs in several important characteristics from the variation of sunspot numbers over the same interval. A spectral analysis approach was used to investigate this variation of  $\bar{A}_p$ , as well as its relation to the variation of sunspot numbers. The data analyzed consisted of the 457 monthly average  $\bar{A}_p$  indices ( $\bar{A}_p$ ) for the interval and the corresponding observed monthly sunspot numbers. Several large samples of the original daily  $\bar{A}_p$  indices were also analyzed. Two of the latter samples covered 11-year intervals (April 1, 1944 - June 2, 1955; and June 1, 1956 - July 31, 1967) and the third covered a 23-year interval (April 1, 1944 - December, 1967).

The  $\bar{A}_p$  spectrum is characterized by seven distinct lines with periods, in order of their relative line amplitudes, of 35.2 and 10.2 years, 6.0 months, 16.1, 7.0, 4.1 and 5.1 years. The lines with periods of 16.1, 7.0 and 5.1 years are not considered to be fundamental; they appear to be either harmonics or side-frequencies of the two strongest lines. The 10.2-year line in the  $\bar{A}_p$  spectrum is matched by a sunspot line at exactly the same period. However, the sunspot cycle precedes the  $\bar{A}_p$  cycle by 18 months. The semiannual variation has maxima close to the equinoxes. The error in the dates of the maxima is of the order of a few days, so the results clearly support the equinoctial hypothesis

for the origin of the semiannual variation. The spectrum of the daily Ap indices is not dominated by the 27-day geomagnetic storm recurrence period, presumably due to the variation of this period over a solar cycle. Apart from an expected solar cycle component and its associated lines, the eight largest lines in the spectrum for the 23-year interval of Ap data occur at periods of 6.0 months, 1.47 years, 27.2 and 27.6 days, 1.23 months, 1.09 years, 13.7 and 14.1 days. There is no evidence for either an annual variation or a variation at the lunar synodic period.

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## I. INTRODUCTION

Variations in solar activity and in the geomagnetic field are of importance in HF radio communications because they are almost always accompanied by fluctuations involving the ionosphere. The relationship between the variations is complex, and has been the subject of numerous studies. In a general sense, the sun provides the driving force for nearly all geomagnetic and ionospheric fluctuations. However, the manner in which it does this varies widely. Some geomagnetic and ionospheric fluctuations result from variations in the intrinsic state of the sun. Solar flare effects and geomagnetic storms are two well-known examples. Other fluctuations result from a variation in the relative orientation or position of the earth with respect to the sun. Seasonal effects and probably also the semiannual variation in geomagnetic activity must originate in this way. Some of the variations are strictly periodic, especially those depending upon solar or planetary motion, whereas others are only approximately periodic (e.g., the solar cycle) or aperiodic (e.g., the occurrence of solar flares and their geophysical effects).

The work reported here is concerned only with periodic or approximately periodic variations of geomagnetic activity, and the object is to identify all significant 'lines' within a broad range of periods, using spectral analysis. A restriction to periods greater than about two days is introduced by the form of the geomagnetic data. There is also an upper limit to the periods for which meaningful spectral data may be obtained. This limit is determined by the length of the interval for which geomagnetic data is available. In the present case, this

limit is of the order of 40 years. For periods greater than about two months comparison is made with the spectrum of sunspot numbers. It is implied throughout that any significant periodicity in geomagnetic activity is likely to have a counterpart in some of the parameters describing the ionosphere. The justification for this assumption is the known existence of strong geomagnetic control over ionospheric motion (e.g., Rishbeth and Garriott, 1964).

There have been two general approaches used in the investigation of geomagnetic fluctuations. One of these involves the use of the original geomagnetic observatory recordings of the magnetic elements (Vestine et al., 1947; Currie, 1966, and other references cited therein) and this approach is undoubtedly the most fundamental. It has the disadvantage that a very great amount of data processing is required, especially if the analysis is to cover an extended interval of time. The other approach, which is used here, is to utilize one of the geomagnetic indices representing geomagnetic activity. This latter approach takes advantage of the ready availability of long intervals of data in digital form and has been used, in particular, by Bartels (1932, 1963) and McIntosh (1959). The two approaches are complimentary but, it should be pointed out, there is a great difference in the physical significance of any observed periodicities. A semiannual variation in the magnitude of the horizontal component of the geomagnetic field, for example, is fundamentally quite different from a semiannual variation in geomagnetic activity, even though the phase and periods may be identical. In the latter case there may be no variation in the average level of the geomagnetic field.

## II. DATA

The quasi-logarithmic Kp index is the best known and most commonly used index of geomagnetic activity. It is intended to give a measure of the average worldwide activity at latitudes below the auroral zones and forms the basis for several other indices. Kp itself is a three-hourly index, and it has a linearized version ap obtained as shown in Table 1. The ap index can be thought of as one half the typical three-hour range of the most disturbed field component at a midlatitude station, and its unit is two gammas ( $2\gamma$ ). In order to indicate daily rather than three-hourly activity, the sum of the eight daily Kp values ( $\Sigma Kp$ ) is frequently used. Since  $\Sigma Kp$  is quasi-logarithmic, a linearized daily index is obtained by averaging the eight daily ap values. The resulting index, Ap, varies within the same range as ap, from 0 to 400 in units of two gammas (the maximum has never been reached), and its use is perhaps preferable in work involving the calculation of averages or in other mathematical procedures (Lincoln, 1967). In the work reported here the Ap index is used exclusively to represent daily geomagnetic activity.

TABLE 1  
Relation between Kp and ap

Kp	0	0+	1-	1	1+	2-	2	2+	3-	3	3+	4-	4	4+
ap	0	2	3	4	5	6	7	9	12	15	18	22	27	32
Kp	5-	5	5+	6-	6	6+	7-	7	7+	8-	8	8+	9-	9
ap	39	48	56	67	80	94	111	132	154	179	207	236	300	400

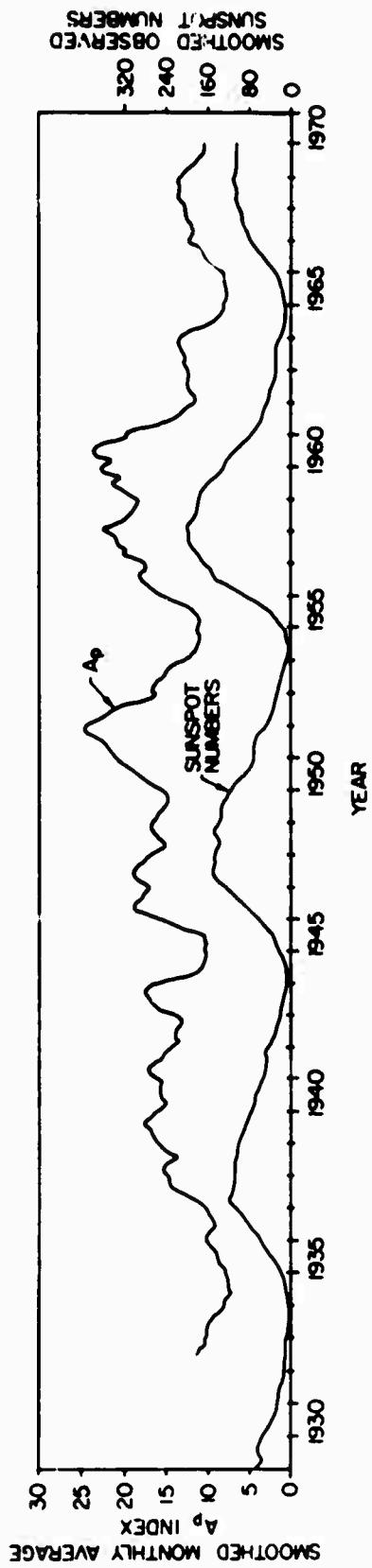


Figure 1. Comparison of the variation of the geomagnetic index  $A_p$  with the variation of sunspot numbers over the interval 1932 to 1970. Smoothed monthly average values are shown.

The auroral electrojet activity index, AE, is not derived from Kp and is thus quite distinct from ap or Ap (Davis and Sugiura, 1966). It is essentially an index of geomagnetic activity within the auroral zones, where the Kp index is not always representative.

Values of Kp and its derivatives have been calculated back to January, 1932 (Bartels, 1962; Solar-Geophysical Data, 1962-1970), and they provide an invaluable record of past fluctuations in the level of geomagnetic activity. Figure 1 compares the variation of Ap over the interval 1932 through 1970 with the corresponding variation of Zürich relative sunspot numbers. The curves were obtained by graphing the smoothed average monthly values of the two variables, using the smoothing formula (Waldmeier, 1961; or Fraser-Smith, 1970a):

$$R'_0 = (R_{-6} + R_6 + 2 \sum_{-5}^5 R_1) / 24$$

where  $R'_0$  is the smoothed monthly value, and where the subscripts on R, the unsmoothed monthly values, vary from -6 through 6 to correspond to the six months before the particular month (negative values), the month itself ( $i = 0$ ), and the six months after the particular month (positive values).

A comparison of the two curves in Figure 1 immediately brings out two distinctive features of the Ap variation. First, the Ap curve is much more irregular than the curve of sunspot numbers. Second, although the general trend of Ap is to follow the variation of sunspot numbers, there are consistently large maxima in Ap during the declining phase of each 11-year sunspot cycle. Both these features of the Ap variation were discussed by Bartels (1963) in an earlier analysis of the time

variations of  $K_p$  and  $A_p$ . The comparatively noisy nature of the  $A_p$  curve is attributed by Bartels, at least in part, to statistical fluctuations resulting from the small fraction of the solar wind sampled by the earth. The solar wind impinging upon the earth's magnetosphere at any particular time must come from a localized region of the sun, and is subject to greater variation in its physical properties than the sunspot numbers, which are integrated over the entire visible disc of the sun. Since  $A_p$  is strongly dependent upon the properties of the solar wind, and upon its magnetic field in particular (see Ballif et al., 1969, for references), it will also be more variable than the sunspot numbers. The peak in  $A_p$  during the declining phase of each sunspot cycle is considered by Bartels to be due to quasi-persistent M-region activity, which is most pronounced in the years just preceding the solar minimum.

The fluctuations in  $A_p$  discussed above tend to mask the extent to which the 11-year periodicity in  $A_p$  follows the sunspot cycle. There is some evidence in Figure 1 that the variation of  $A_p$  may lag a little behind the sunspot variation, but the extent of the lag is impossible to determine from the curves alone. The noisy nature of the  $A_p$  curve could also easily mask small but otherwise significant periodicities, which may have no connection with sunspots or possibly even with the sun itself. An example of a variation that has no counterpart in the variation of sunspot numbers is provided by the well-established semiannual variation of the geomagnetic field (Bartels, 1963; Currie, 1966, for example). This variation is so strong that it can generally be observed with little difficulty above the background fluctuations in  $A_p$ . In Figure 1, however, the smoothing process has virtually eliminated all periods less than 12 months, and the semiannual variation is not visible.

The powerful techniques of spectral analysis developed over the last decade, and based upon the Fast Fourier Transform (FFT), make possible a detailed search for periodicities in the time variation of Ap and other indices. In this paper we report the results of an extensive spectral analysis of Ap, using the 457 monthly average Ap indices ( $\bar{A}_p$ ) in the 38-year interval January, 1932 through January, 1970 (Table 2). For comparison with the results for  $\bar{A}_p$ , the spectrum of the average monthly sunspot numbers was calculated for the same 38-year interval. Two separate 11-year samples of the daily Ap indices (April 1, 1944 through June 2, 1955, i.e., 4080 daily values; and June 1, 1956 through July 31, 1967; i.e., 4078 daily values) were also analyzed. They were then combined, with some additions, into a single large 23-year sample (April 1, 1944 through December 31, 1967, i.e., 8674 daily values) and reanalyzed.

TABLE 2

Monthly Average Ap Indices ( $\bar{A}p$ ) for the Interval  
January, 1932 through December, 1970

Year	JAN.	FEB.	MCH.	APL.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
1932	11.0	11.6	16.3	17.3	14.6	6.6	6.8	11.7	12.1	10.1	8.3	9.1
1933	9.6	11.3	12.4	12.5	11.8	8.5	7.5	9.2	11.7	10.1	9.3	7.2
1934	5.9	8.0	10.9	6.1	6.6	5.3	5.9	9.1	10.2	5.9	4.7	8.3
1935	3.7	9.8	9.6	6.0	6.1	9.5	6.6	5.4	13.0	12.3	8.3	9.3
1936	9.0	10.9	6.9	14.8	10.2	11.9	10.5	4.9	4.8	8.6	9.6	5.5
1937	7.2	12.6	11.9	20.0	12.8	12.0	12.2	10.1	9.1	20.0	11.5	9.7
1938	28.5	15.9	13.2	18.3	17.6	8.9	13.2	12.4	16.8	16.4	10.0	11.4
1939	7.1	15.3	19.1	28.0	21.4	15.1	19.0	19.3	13.2	21.7	8.7	11.0
1940	14.9	12.5	36.6	11.0	13.4	16.4	11.7	10.8	14.0	14.2	16.0	14.6
1941	14.1	17.6	32.8	15.0	10.8	10.9	19.3	16.3	26.7	11.3	16.1	11.3
1942	9.1	11.8	22.4	17.1	7.8	8.0	12.6	12.7	16.9	22.2	14.6	10.6
1943	10.8	9.3	13.0	14.1	14.5	12.4	14.6	31.2	25.4	23.5	20.2	14.1
1944	12.7	11.8	17.3	15.1	9.4	7.6	5.7	9.1	10.3	11.2	6.1	13.6
1945	10.2	9.6	16.8	13.1	9.0	8.8	9.4	7.2	9.7	11.3	7.6	13.1
1946	12.6	22.2	33.5	19.7	17.6	16.0	22.0	11.3	34.3	13.1	12.5	9.1
1947	12.5	11.8	32.2	17.7	14.0	16.0	16.0	25.3	32.2	23.1	14.1	10.6
1948	11.6	12.6	17.4	12.6	16.6	9.9	9.9	20.0	14.9	27.1	16.0	13.0
1949	20.3	14.3	19.2	14.3	16.5	14.1	7.7	14.2	12.7	25.3	15.5	8.7
1950	11.6	17.7	14.2	16.3	16.2	14.2	14.0	24.6	22.3	27.9	19.9	15.5
1951	15.9	21.8	21.3	27.2	20.5	17.4	19.9	21.6	39.3	24.1	18.0	20.0
1952	19.4	26.0	33.4	33.6	26.9	17.5	14.8	12.7	22.9	20.0	12.4	15.1
1953	15.4	15.0	20.9	15.5	15.9	12.8	15.5	18.7	21.1	16.3	13.5	6.9
1954	8.7	16.4	16.2	14.0	7.4	5.8	7.6	9.9	16.6	14.8	8.6	6.4
1955	12.1	11.6	14.4	13.9	11.4	9.5	6.0	9.1	12.7	11.3	13.1	8.1
1956	17.6	15.4	20.2	27.4	25.5	16.7	13.0	15.4	17.6	13.5	24.0	10.3
1957	16.7	16.6	26.0	21.5	10.7	22.1	15.8	13.6	49.3	13.8	18.0	17.7
1958	14.7	26.9	26.3	19.6	17.5	24.1	24.8	17.7	20.5	16.3	7.8	15.6
1959	13.6	24.0	24.1	16.6	18.9	15.4	31.9	23.4	28.1	18.9	21.8	19.2
1960	14.9	14.0	16.3	41.6	24.2	19.7	20.1	20.1	20.5	36.1	32.5	21.4
1961	11.8	15.6	14.0	14.0	13.3	13.8	27.7	10.6	12.6	16.3	9.9	12.2
1962	7.0	10.5	8.3	14.0	7.2	9.3	11.5	14.9	18.9	20.0	12.8	12.8
1963	10.5	8.6	8.3	9.6	11.1	10.6	11.9	13.5	28.5	15.4	12.3	10.5
1964	11.8	11.9	12.7	13.3	10.5	8.6	9.2	8.0	11.1	9.6	7.3	5.3
1965	6.2	9.3	6.3	7.9	5.6	9.7	7.7	8.5	10.1	6.7	6.8	7.1
1966	7.3	8.2	12.6	6.6	9.2	6.5	8.9	11.2	21.0	10.1	9.5	11.6
1967	11.4	11.3	7.4	8.9	25.4	11.9	8.0	8.9	16.3	9.9	10.3	14.1
1968	11.5	16.2	13.4	13.1	13.3	16.8	10.2	11.7	13.8	15.5	17.0	9.9
1969	8.4	14.7	17.0	14.1	17.1	9.1	7.9	8.0	15.0	8.7	9.9	6.8
1970	7.3	6.8	17.5	15.6	9.1	10.5	19.0	13.4	10.8	12.5	11.9	8.9

### III. METHOD OF ANALYSIS

The geomagnetic and sunspot data were spectral analyzed using a version of the FFT on an IBM 360/67 computer. The program required input data that were both evenly spaced and whose number ( $N$ ) was a power of two (less economical programs exist for spectral analysis where the latter restriction does not apply). The basic computer output consisted of a list of periods and their associated amplitudes and phases. The spacing between the periods could be decreased by the standard interpolation technique of adding zeros to the end of the original dataset before analysis. It was necessary, in fact, to add zeros to the end of the original data in all cases, because of the restriction of the number of data to a power of two.

If the number of input data was  $N$ , including zeros, the periods for which amplitude and phase were calculated were given by  $N/n(N)$ , where  $n(N)$ , which will be called the period index, was an integer varying from 0 to  $(N/2 - 1)$ . The units of period were determined by the spacing of the input data. Thus an input of monthly values gave output periods measured in months. The output for  $n(N) = 0$  was anomalous and consisted of the average value of the input data. To avoid a large spectral peak at this value of  $n(N)$ , and the accompanying interference from its finite width and sidelobes, the original data were always averaged and the average subtracted from each value of the data before zeros were added and the dataset analyzed. This ensured that the output for  $n(N) = 0$  was always zero or very close to that value. The longest period in the analysis was thus given by  $n(N) = 1$ . The shortest period for which an output could be obtained was given by  $n(N) = (N/2 - 1)$ , i.e., a period

just greater than twice the effective sampling period. For 4096 monthly values the longest period was 4096 months or 341.3 years, and the shortest period was 4096/2047 or 2.001 months.

A Hamming window was used in some of the earlier stages of the analysis. It successfully reduced sidelobe levels associated with the spectral lines. However, its use created problems at the lowest frequencies and the unavoidable widening of the spectral lines was also considered a disadvantage. The Hamming window was therefore discontinued and the results presented here were all obtained with the square window characteristic of an unmodified finite interval of data.

Once a set of data was analyzed, the spectrum was obtained by graphing amplitude against the period index  $n(N)$ . Individual spectral lines, if any, were then identified and their amplitudes, period indices and phase found. For each line these data determined a cosinusoid of the form

$$(2AN/N_0) \cos [2\pi n(N)I/N + P]$$

where  $A$  is the line amplitude,  $P$  the phase,  $N_0$  the number of the original data (before the addition of zeros), and  $I$  a time variable with the same units as the spacing in the input data. Note that  $I = 0$  was chosen to correspond in time with the first point of the original data. Each of the cosinusoids given by the above formula represents a periodic component in the original data, and the spectrum analysis was considered to be complete when all major cosinusoids had been determined.

TABLE 3

Details of periodicities in the variation of the monthly average values of  $\bar{A}p$  ( $\bar{A}p$ ) and Zürich relative sunspot numbers for the interval January 1932 through January 1970. The units of amplitude are the same as those for the variables.

(a)  $\bar{A}p$   
(Overall Average Value = 14.4)

Relative Size	Period	Amplitude	Phase
1	35.6 yrs.	3.71	155°
2	10.2 yrs.	3.46	83°
3	6.00 mos.	2.98	228°
4	16.1 yrs.	2.41	127°
5	7.04 yrs.	1.57	38°
6	4.10 yrs.	1.55	67°
7	5.14 yrs.	1.39	19°

(b) Monthly Average Sunspot Numbers  
(Overall Average Value = 71.1)

Relative Size	Period	Amplitude	Phase
1	10.2 yrs.	70.1	136°
2	44.1 yrs.	20.5	162°
3	16.2 yrs.	20.3	156°
4	5.30 yrs.	19.4	38°
5a*	6.5 yrs.	9.8	65°
5b*	7.2 yrs.	8.5	99°

\* This line is a doublet and there is some evidence of other smaller lines.

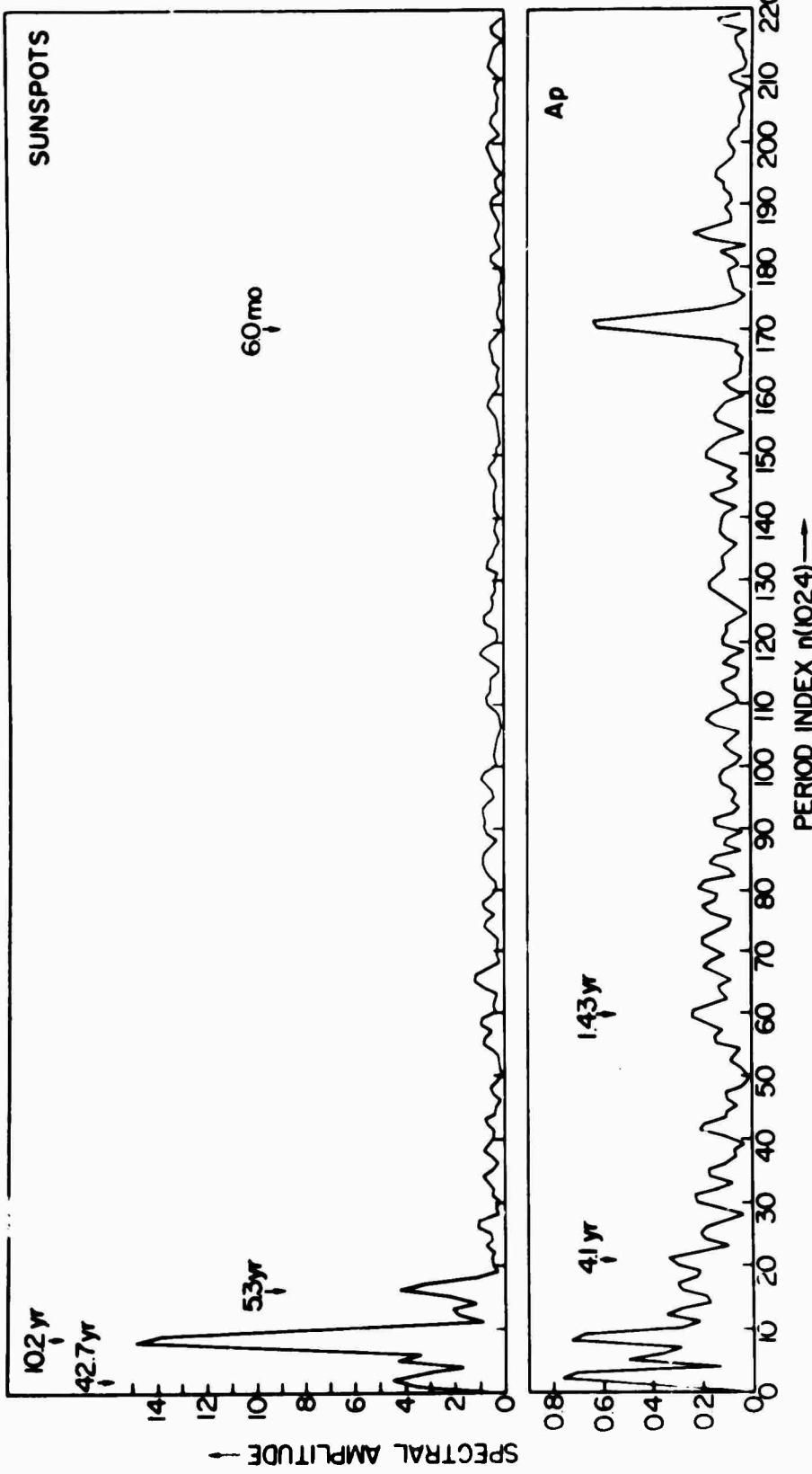


Figure 2. Comparison of the spectrum of monthly average sunspot numbers (top) with the spectrum of monthly average Ap indices (bottom) for the interval January, 1932 through January, 1970 (a total of 457 months). The periods in months are obtained from the period index  $n(1024)$  by the use of the formula: period =  $1024/n(1024)$  months. Thus the period decreases to the right in the figure.

#### IV. RESULTS

##### A. SPECTRA of $\bar{A}_p$ AND SUNSPOT NUMBERS

The computed amplitude spectra for the 457  $\bar{A}_p$  values, January 1932 through January 1970, and for the corresponding monthly average sunspot numbers are shown in Figure 2. Unsmoothed values were used in both cases for the spectral analysis. The spectrum for  $\bar{A}_p$  has seven comparatively distinct lines whose periods, phases and equivalent cosinusoidal amplitudes (i.e.,  $2AN/N_0$ ) were all accurately determined using a 4096-point analysis. These data are listed in Table 3. Period, phase and amplitude data for the five identifiable lines in the sunspot spectrum are also given in Table 3. There are some small lines present in the  $\bar{A}_p$  spectrum that may have some significance and which are not included in the table. The line at a period of 1.43 years is an example. These smaller lines gradually merge into the noise background (which is clearly more pronounced for  $\bar{A}_p$  than for the sunspot numbers) and for period indices greater than  $n = 220$ , at which the display in Figure 2 is terminated, there are no further significant lines in either the  $\bar{A}_p$  or sunspot spectrum.

The width of any spectral line shown in Figure 2, and in later figures, is proportional to  $N/N_0$ . If the width is measured between the half-amplitude (6 db) points of the characteristic  $(\sin x/x)$  pattern for each line the width may be shown to be  $1.2 N/N_0$ . Thus, in Figure 2 the half-amplitude width is given by  $\Delta n(1024) \approx 2.7$ . For a given initial value of  $N$  we have the important property that an increase in the size  $N_0$  of the original non-zero dataset gives greater resolution. In the

spectra shown the lines may not always appear as narrow as required by the above formula. This is caused by the presence of adjacent lines or by noise.

The 11-year solar cycle variation of  $\bar{A}p$  seen in Figure 1 is distinctly weaker than the same variation of sunspot numbers; it is not surprising therefore that the amplitudes of the solar cycle lines in Figure 2 have the same relationship. It is noteworthy that the computed periods for these lines are both exactly the same (10.2 years). However, there is a considerable difference in phase which results in the sunspot cycle leading the  $\bar{A}p$  cycle by very nearly 18 months. There are no other lines where the  $\bar{A}p$  and sunspot periods match exactly, although the 16.1-year and 5.1-year lines in the  $\bar{A}p$  spectrum have obvious counterparts with periods of 16.2 years and 5.3 years in the sunspot spectrum. In both cases the indicated sunspot cycle leads the  $\bar{A}p$  cycle, but not by as much as 18 months: the 16-year cycle leads by approximately 16 months and the 5-year cycle by approximately 3 months.

There are other interesting differences between the  $\bar{A}p$  and sunspot spectra. Perhaps the most noticeable of these differences is the absence of matching sunspot lines for the 6-month and 4-year lines in the  $\bar{A}p$  spectrum. Both periodicities must be generated in the vicinity of the earth. There also appears to be little correspondence, at the lowest frequencies, between the 35-year line in the  $A_p$  spectrum and the 44-year line in the sunspot spectrum. However, for these long periods the spectrum is comparatively inaccurate and there may well be better agreement when a greater interval of  $\bar{A}p$  data is available. Note the presence of the long periods in Figure 1.

Geomagnetic fluctuations have been reported at periods of 12-months (i.e.,  $n(1024) = 85.3$ ) and 26-months (i.e.,  $n(1024) = 39.4$ ). Cynk (1941), Vestine et al. (1947) and Currie (1966) found the annual variation in standard geomagnetic records, while McIntosh (1959) detected a similar but small variation in certain K indices. There is less evidence for the 26-month, or quasi-biennial, line (Currie, 1966). A 26-month line might be expected, however, because Shapiro and Ward (1962) reported a small peak in the spectrum of sunspot numbers with a period of 25.7 months (also see Wescott, 1964) and there is evidence of stratospheric wind oscillations with similar periods (Reed et al., 1961). Thus, it is pertinent that there is no trace of either a 12- or 26- month line in the sunspot and  $\bar{A}p$  spectra shown in Figure 2. A moderately sharp peak occurs at a period of 24.8 months, in the  $\bar{A}p$  spectrum, but its amplitude is still too small for it to be considered significant. The absence of a 12-month period in  $\bar{A}p$  has been considered to be the result of the procedure involved in computing the planetary index (McIntosh, 1959; Priester and Cattani, 1962) and this view may be substantially correct. Nevertheless, the earth as a whole receives  $\sim 6\%$  more energy from the sun in early January (perihelion) than it does during early July (aphelion). It would be reasonable to expect this world-wide effect to influence a planetary index such as  $A_p$ .

There are several different ways in which the spectral line data given in Table 3 can be in error. When the lines are well separated or isolated, as in the case of the 6-month line, the only major source of error is the presence of noise. In this case the error can be estimated quite accurately. When the lines are closely spaced, the side-

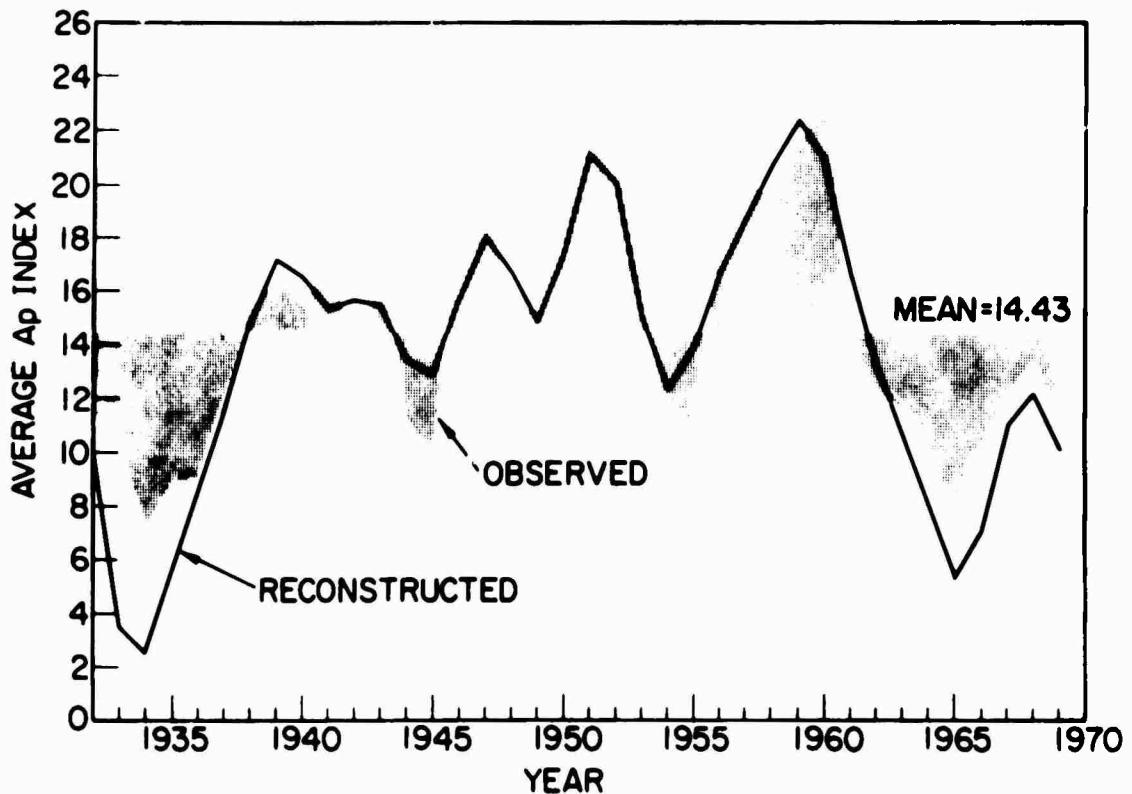


Figure 3. Comparison of the observed and reconstructed variations of the Ap index over the interval 1932 - 1970. Average yearly values of the Ap index are shown in each case. The reconstructed variation was obtained using the period, amplitude and phase data for the seven most prominent lines in the spectrum shown in Figure 2.

frequencies (sidelobes) accompanying each line become superimposed upon some of the other lines and introduce period, phase and amplitude errors. This effect is particularly marked when a very strong line is present. The close spacing of the long-period lines in both the  $\bar{A}p$  and sunspot spectra implies the presence of some error from this source. At the lowest frequencies the lines are distorted from their  $(\sin x/x)$  pattern by the sidelobes of their negative frequency counterparts. This introduces an additional source of error. The overall effect of noise and sidelobe errors is difficult to estimate. In order to show how well the data in Table 3 represent the actual observed variation, a reconstructed  $\bar{A}p$  variation was calculated using the data in the table (the average value of  $\bar{A}p$  was also included). The reconstructed variation is graphed in Figure 3, using yearly averages to give a compact figure, and compared with the original variation of  $Ap$ . Considering that only seven cosine series were used in the reconstruction, the agreement can be considered very good.

#### B. SEMIANNUAL VARIATION

It has been known for a number of years that there is a semiannual (6-month period) fluctuation in geomagnetic activity [e.g., Bartels, (1932, 1963), Vestine et al. (1947), McIntosh (1959), Ward and Shapiro, (1961)]. Two theories have been advanced to explain the fluctuation. They are commonly referred to as the 'axial' and 'equinoctial' hypotheses, and have most recently been discussed by Currie (1966), Shapiro (1969) and Boller and Stolov (1970). The axial theory relates the semiannual variation to changes in the heliographic latitude of the earth, with maximum activity occurring just after the times of maximum ( $+7.2^\circ$ ) and minimum ( $-7.2^\circ$ )

latitude, i.e., September 7 and March 6, respectively. The equinoctial hypothesis differs in that it relates the semiannual variation to changes in the earth's inclination to the earth-sun line, with maximum activity occurring at the equinoxes.

A 6-month line is a prominent feature of the spectrum shown in Figure 2. In view of the two competing theories to explain its origin, it is pertinent that phase and period data obtained for the line define maxima which occur either on or very close to the equinoxes\*. The period index obtained for the 6-month line, using the 4096 point analysis, is  $n(4096) = 682.3$ , corresponding to a period of 6.0032 months. The phase for this value of  $n(4096)$  is  $228^\circ$ . Both period index and phase are obtained accurately by using a simple graphical interpolation process. Together they give a March maximum that varies from March 21, 1932 ( $I = 2.20$ ) to March 29, 1970 ( $I = 458.5$ ) over the time interval defined by the original dataset. The September maximum has a similar variation in time. It may reasonably be expected that the 6-month line is contaminated by an element of noise and that the correct period is exactly six months, i.e.,  $n(4096) = 682.67$ . The phase corresponding to this period is  $221^\circ$ . These values of period and phase give maxima on March 25 and September 25. The isolation and strength of the 6-month line (Figure 2) make possible an accurate estimate of the error in the dates of maximum

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\* Note. A preliminary presentation of the results in this report was made at the National Fall Meeting of the American Geophysical Union, December 1970 (Fraser-Smith, 1970b). At that time integer values of the index were thought to apply to the beginning and not the center of a month. This incorrectly gave semiannual maxima on or near March 9 and September 9.

activity. A noise signal of the same amplitude as the average background level in Figure 2 introduces, at the most, a phase error equal to 4.5 days in these dates. We may therefore conclude that the maxima of the 6-month variation, occur on March ( $25 \pm 3$ ) and September ( $25 \pm 3$ ). Thus, the results of this spectral analysis argue strongly for the equinoctial hypothesis.

As indicated in Table 3, the amplitude of the semiannual variation is 2.98 in the same units as  $Ap$ , or approximately  $6.0\gamma$  at a typical midlatitude station. A small correction factor of 1.047 must be applied to this amplitude as compensation for the  $(\sin \chi/\chi)$  variation accompanying the use of average monthly values (the correction factors for the other amplitudes in Table 3 are all negligible). Thus, the amplitude of the semiannual variation in geomagnetic activity at a typical midlatitude station is of the order of  $6.2\gamma$ . The amplitude in terms of  $Ap$  is in good agreement with results published by Bartels (1963) and Wilcox (1968), for example. Boller and Stolov (1970) quote a value of  $7.5\gamma$  (range  $\sim 15\gamma$ ) for the amplitude of the semiannual variation which is also in good agreement with the  $Ap$  results.

### C. SPECTRUM OF $Ap$

#### 1. First Interval: June 1, 1956 - July 31, 1967

Figure 4 shows part of the computed amplitude spectrum for the 4078 daily values of  $Ap$  in the 11-year interval June 1, 1956 through July 31, 1967. The high frequency end of the spectrum actually extends out to  $n(4096) = 2047$ , corresponding to a period of very nearly 2 months, but it contains few distinct lines beyond  $n(4096) = 310$  and

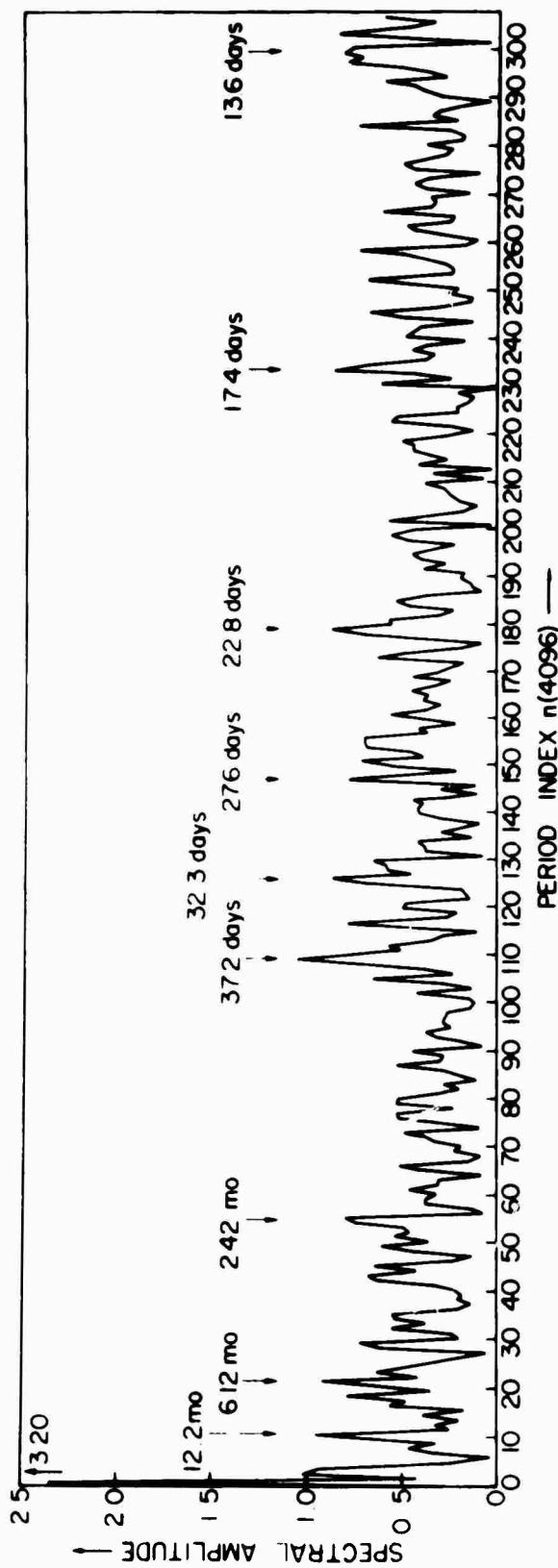


Figure 4. Spectrum of the daily Ap indices for the 11-year interval June 1, 1956 through July 31, 1967 (a total of 4078 days). The periods in days are obtained from the period index  $n(4096)$  by use of the formula: period =  $4096/n(4096)$  days. Thus the period decreases to the right in the figure.

has been omitted from the figure. There is a considerable difference in appearance between the spectrum of Ap (Figure 4) and the  $\bar{A}p$  spectrum shown in Figure 2. Some of this difference is caused by the new data, but there is also an increase of over two in the resolution, which gives the Ap spectrum a more jagged appearance. The half-amplitude width of any line is now given by  $\Delta n(4096) \approx 1.2$ .

Table 4a gives a listing of the periods and amplitudes of the major lines. The longest periods are quite approximate and the values shown in the figure are mostly applicable to those integral values of the period index with the largest spectral amplitude. It is possible, in principle, to obtain the periods and phases more precisely by adding zeros to the original dataset and then repeating the analysis. However, for these daily data, a significant increase in precision at the long periods requires a final dataset that is larger than it is practical to analyze. Another difficulty is the close spacing of the lines at the long periods. The inaccuracies introduced by overlapping cannot be eliminated by the interpolation procedure just described. Greater resolution is required for this purpose, i.e., more daily data. It was to avoid the necessity of analyzing a huge dataset that monthly average data were used in the investigation of the long periods.

An inspection of Figure 1 shows a well-defined single cycle of solar activity during the 11-year interval of data under analysis. This accounts for the large amplitude line at  $n(4096) = 1$ . The second strongest line has a period of 4.2 years but, as explained above, the period is quite approximate and the line may well be the second harmonic of the 11-year line. The third strongest line has a period of 37.2 days, and its origin is not clear at the present time. However,

TABLE 4

Details of periodicities in the variation of Ap over the intervals  
(a) June 1, 1956 through July 31, 1967 (average value of Ap = 15.0) and  
(b) April 1, 1944 through June 2, 1955 (average value of Ap = 16.1). The  
units of amplitude are the same as those of Ap. There is an error of  
the order of 1% in periods less than 6 months.

(a)

Relative Size	Period	Amplitude	Phase
1	~ 11 yrs	6.4 ( $\pm 0.2$ )	266° ( $\pm 20^\circ$ )
2	~ 4.2 yrs	2.2	275°
3	37.2 days	2.1	230°
4	~ 12 mos	2.0	345°
5	~ 6.0 mos	1.9	202°
6	22.8 days	1.8	260°
7	32.3 days	1.8	262°
8	17.4 days	1.8	235°
9	13.4 days	1.7	113°
10	13.6 days	1.6	305°

(b)

Relative Size	Period	Amplitude	Phase
1	~ 10 yrs	4.2 ( $\pm 0.2$ )	100° ( $\pm 20^\circ$ )
2	~ 3 mos	3.7	55°
3	27.2 days	2.3	170°
4	14.1 days	2.1	210°
5	29.2 days	2.0	290°
6	30.1 days	2.0	4°
7	25.3 days	1.95	52°
8	13.7 days	1.9	85°
9	~ 1.4 yrs	1.9	165°
10	1.77 mos	1.7	95°

while it is the strongest line during the maximum phase of the Ap cycle (January 1, 1956 - December 31, 1961; average Ap = 19.46), it is lost in the noise background during the minimum phase (January 1, 1962 - December 31, 1967; average Ap = 10.79). Thus, for the interval June 1, 1956 through July 31, 1967, the 37.2 day period appears to be a feature of a high average level of Ap.

It is perhaps surprising that the 27-day geomagnetic storm line is so weak in Figure 4, but this is caused by the variation of the storm recurrence period over the duration of a solar cycle, as well as by a lack of coherence in the storm induced variations. Some evidence for a variation in period is provided by separate spectra for the six years of maximum Ap activity and for the six years of minimum activity. During the minimum phase two closely space lines with periods of 27.1 and 27.6 days are in evidence. They are the second and third strongest lines, respectively, in the spectrum of Ap for the minimum phase. During the maximum phase, on the other hand, the strongest storm line occurs at a period of 26.3 days, and there is a second weaker line at a period of 28.8 days. There is no 27.0-day line and only an insignificant 27.7 day line.

## 2. Second Interval: April 1, 1944 - June 2, 1955

The computed amplitude spectrum for the 4080 daily values of Ap in the 11-year interval April 1, 1944 through June 2, 1955 differs little in general form from the spectrum shown in Figure 4. However, the details of the major lines contain numerous differences from those for the interval June 1, 1956 through July 31, 1967, (see Table 4b). In particular, the 6-month line is much more prominent, the 37.2-day

TABLE 5

Details of periodicities in the variation of Ap over the interval April 1, 1944, through December 31, 1967. The average value of Ap over the interval is 15.4 and the units of amplitude are the same as those of Ap. Periods less than 6 months may be in error by  $\pm 1$  in the third significant figure. The estimated error in the 6 month period is twice as large.

Relative Size	Period	Amplitude	Phase
1	6.00 mos.	3.15( $\pm 0.10$ )	14°( $\pm 10$ °)
2	1.47 yrs.	1.62	220°
3a	27.2 days	1.59	203°
3b	27.6 days	1.50	65°
5	37.4 days	1.53	279°
6	1.09 yrs.	1.51	18°
7	13.7 days	1.40	285°
8	14.1 days	1.32	221°
9	18.7 days	1.30	225°
10	30.5 days	1.29	95°
11	54.0 days	1.27	130°
12	13.6 days	1.25	278°
13	26.9 days	1.23	77°
14	9.39 days	1.23	290°

line is insignificant and there is a more orderly grouping of lines about the 27.3-day geomagnetic storm recurrence period.

3. Third Interval: April 1, 1944 - December 31, 1967

The 23-year interval of daily Ap data, April 1, 1944 through December 31, 1967, was analyzed using a 16384 point analysis. Table 5 gives a listing of the periods, amplitudes and phases of the fourteen strongest lines with periods less than two years and greater than two days (the longest periods are omitted because their characteristics are more accurately described in the analysis using monthly average data). The fourteen lines have amplitudes that vary from much greater than twice the average background level to values slightly less than twice the average background.

The amplitude and phase data in Table 5 for the 6-month line give a semiannual variation that is in excellent agreement with the variation obtained by using monthly average data. The times of maximum derived from the daily data are March and September 24, differing by only one day from the times obtained with the monthly data, and there is almost exact agreement between the calculated amplitudes. However, the estimated errors are larger for the daily data, and there is an uncertainty of at least  $\pm 5$  days in the times of maximum.

There was some indication of a 1.4-year line in the spectrum of Figure 2 (monthly data) and it is a more obvious feature in the spectrum for the daily Ap data. An unexpected line occurs at a period of 1.09 years, with an amplitude not much smaller than that of the 1.43-year line. These two lines are all that are evident with periods greater than six months. Again, there is no trace of an annual line.

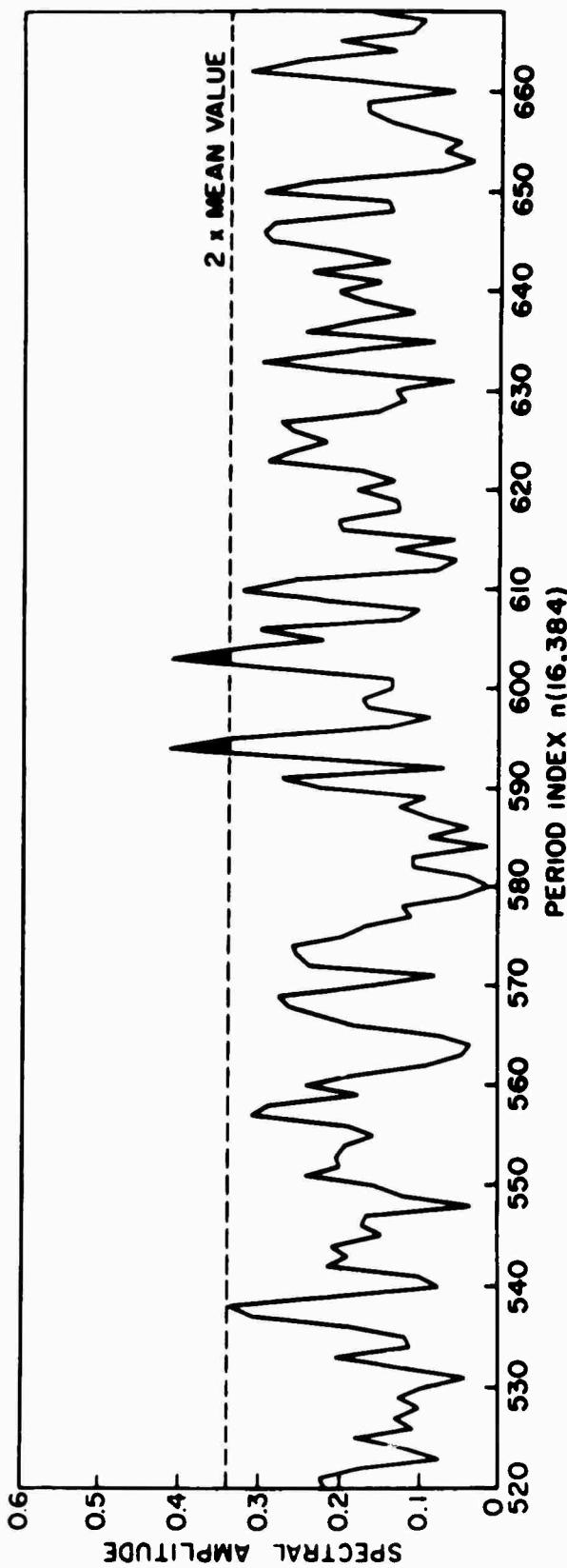


Figure 5. A part of the spectrum of the daily Ap indices for the 23-year interval April 1, 1944 through December 31, 1967 (8674 daily values). The section of the spectrum shown covers the range of solar rotation periods. Periods in days are obtained from the period index  $n(16384)$  by use of the formula: period =  $16384/n(16384)$  days. Thus  $n(16384) = 520$  corresponds to a period of 31.63 days and  $n(16384) = 660$  to a period of 24.90 days. The indicated mean value applies only to the amplitudes shown in the figure.

All other lines in the spectrum can be considered to be related in some way to solar rotation or to the 37-day line previously noted in Figure 4. The few distinct lines with periods less than the solar rotation period range all appear to be harmonics of longer period lines. The 18.7-day line is obviously the second harmonic of the 37-day fundamental and the weak 9.39-day line may also be harmonic, although there is no intermediate 12.5-day line. It is interesting that the period range 2-9 days is almost completely quiet.

Figure 5 shows the spectrum in the vicinity of the solar rotation period, which has a mean value, derived from sunspot observations, of 27.27 days. A very interesting result of this analysis is the existence of a clearly-resolved spectral doublet, with periods of approximately 27.2 and 27.6 days. The amplitudes of these two lines are equal within estimated error, and are more than twice the average amplitude of the local background. A similar pair of lines was observed in the spectrum of Ap for the minimum phase of the cycle 1956-1957 (see section 1 above). Other lines near the doublet are all notably smaller in amplitude. The other short-period line (14.1 days) is more difficult to explain. It is tempting to ascribe it to a 28.2-day fundamental ( $n(16384) = 581$ ), a hypothesis which could also account for the 9.39-day line. However, there is no evidence for such a line.

Finally, it is of some interest that there is no trace of a line for  $n(16,384) = 554.8$ , which corresponds to the 29.53-day synodic period of the moon. There have been many attempts to resolve this line, but no conclusive result has been achieved (see Fraser-Smith, 1969, for references).

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## V. DISCUSSION

The analysis described in this report confirms that most periodic changes in geomagnetic activity with periods greater than two days are closely related to changes associated with the sun, either through the solar cycle (period  $\sim 10$  years) or through solar rotation (period  $\sim 27$  days). There are a few exceptions. The best example is the comparatively large semiannual variation, but there are other non-solar related variations of smaller amplitude that are not generally discussed in the literature. Details of these lines constitute some of the points of interest arising from this work. These points of interest will now be summarized as follows:

- (1) There is an exact correspondence in period (10.2 years) between the solar cycle variations of sunspot numbers and the Ap index. However, the Ap cycle lags behind the sunspot cycle by 18 months.
- (2) The Ap spectrum has a 4.1-year line that is not matched by a line in the sunspot spectrum.
- (3) The Ap spectrum also has weak lines at periods of 1.43 and 1.09 years that are probably not related to anything connected with the sun. (There are no lines in the sunspot spectrum at either period.)
- (4) Neither an annual line nor a quasi-biennial line (26-month period) appear in the Ap spectrum.
- (5) The semiannual variation in Ap has an amplitude of  $\sim 3.1$  (in units of Ap; equivalent to  $\sim 6.2$  at a standard station) and maxima that lie within three days of the equinoxes. Thus, the equinoctial hypothesis for the origin of this variation is favored.
- (6) A small line exists at twice the solar rotation period i.e., with a period of 54 days.
- (7) There exists a 37-day line, and at least one of its harmonics, in the Ap spectrum.

- (8) The 29.53-day synodic period of the moon has no detectable counterpart in geomagnetic activity.
- (9) For periods in the vicinity of the average solar rotation period (27.3 days) the Ap spectrum is dominated by a doublet with periods of 27.2 and 27.6 days. There is no line at the average solar rotation period.
- (10) There is a weak 14.1-day line with no corresponding fundamental of  $\sim$  28 days period in the solar rotation period range.
- (11) The period range of 2-9 days contains no significant lines and, with the possible exception of the 14.1-day line, there are probably no significant fundamental periods in the range 9-26 days.

The complex structure at the lowest periods in the spectrum of the monthly average Ap indices (Figure 2) can be explained almost entirely as side-frequencies and harmonics accompanying two fundamental periods: a very long period of  $\sim$  35 years and the 10-year solar cycle period. For a 10.2-year fundamental and a 457 month interval of data the first five of the shorter sidelobe periods are 7.4, 6.15, 5.3, 4.6 and 4.1 years. Assuming the fundamental has an amplitude of 1.00, the amplitudes of the five sidelobes are 0.22, 0.13, 0.09, 0.07 and 0.06, respectively. In the case of the observed 7- and 6- year lines the sidelobe periods and amplitudes are sufficiently close to the predicted values to give a complete explanation for the lines. For the 5-year line, however, the observed amplitude is over three times greater than the predicted sidelobe amplitude, in both the sunspot and Ap spectra. This line clearly indicates the presence of the 5.1-year second harmonic of the 10.2-year fundamental period. No lines are observed at a period of 4.6-years in either the sunspot or Ap spectra and, as already noted, there is no 4.1-year line in the sunspot spectrum. Thus, it is surprising that there should be a distinct 4.1-year line in the Ap spectrum.

The amplitude of this line is about eight times greater than the predicted sidelobe level at a period of 4.1 years. Also, a period of 4.1 years is not part of the harmonic sequence for a 10.2-year fundamental period. The only conclusion is that the 4.1-year line must be generated in the earth's environment.

The 16-year line in both the sunspot and  $\bar{A}_p$  spectra may be explained as either the first lower-period sidelobe of the strong 10.2-year line, or as the second harmonic of the 35-44 year low-period lines. In fact, it probably involves the superposition of both a sidelobe and an harmonic.

In an earlier analysis of the spectrum of geomagnetic activity, as represented by the international magnetic character figure Ci, Shapiro and Ward (1966) found three peaks with periods near 27 days. The periods of the three peaks were 27.4, 29.4 and 26.0 days, in order of their relative magnitudes, and the spectral resolution in period was about one-half day for that part of the spectrum. The resolution achieved was not quite sufficient to resolve the doublet shown in Figure 5. As illustrated by the latter figure, much greater spectral resolution is now possible: The half-amplitude width of any spectral line in the figure is given by  $\Delta n(16,384) \approx 2.3$ , which is equivalent to about 0.11 day near a period of 27 days. Even this resolution could be improved upon by the use of a larger sample of original data.

A period of 29.4 days is equivalent to  $n(16,384) = 557$  in Figure 5, while  $n(16,384) = 630$  corresponds to a period of 26.0 days. There are small peaks near each period index but their amplitudes are at least ten per cent less than twice the mean background level and they are probably not significant.

Doublets and higher order groupings of lines are frequently observed in spectra and they are often produced by a splitting process. A good example is the Zeeman effect. It is therefore interesting to see if the doublet in Figure 5 could be produced by splitting from a single original line. If amplitude modulation is involved there should be three lines. That is, an original line of undiminished amplitude and two side-frequency lines with amplitudes depending upon the strength of modulation. Although there is no apparent center line in Figure 5, we will assume the doublet consists of two side-frequencies. Then

$$fc - fm = 1/27.6 \text{ cycles per day}$$

$$fc + fm = 1/27.2 \text{ cycles per day}$$

where  $fc$  is the center frequency and  $fm$  is the modulation frequency.

Solving for  $fm$  and converting to period we obtain a modulation period of 10.3 years.

There is clearly some reason to believe that the solar radiation line is modulated by the solar cycle. The absence of the center line suggests frequency modulation, but the physical process is hard to visualize. A simpler explanation is that the center line is reduced by noise to about half its intrinsic amplitude. However, a doublet was also observed in a much smaller subset of the data that was used to give Figure 5, and it is unlikely that the center frequency would be equally affected by noise in the two cases. There are sidelobe frequencies accompanying each of the doublet lines and it is possible that a combination of noise and the doublet sidelobes leads to the reduction of the center line. Note that the second sidelobes of the doublet lines occur at periods of 27.41 and 27.38 days.

A 56-day line in the spectrum of Ap suggests that some of the active regions on the sun may diminish in strength and then increase again with a period of the order of twice the average solar rotation period. The line is weaker than the 27.2- and 27.6-day doublet, but its amplitude is at least as great as twice the average background level. Its period also suggests that it is significant.

The ionosonde technique for studying the ionosphere was developed in the 1930's and data obtained from ionosondes has been used in numerous systematic studies of the ionospheric variations. Geomagnetic control of the ionosphere was first detected by the use of ionosonde records (Bailey, 1948) and the seasonal variation of  $f_o F2$  has since been investigated and widely discussed (Mayr and Mahajan (1971) and references therein). The two major seasonal effects are the "winter anomaly", where the winter  $f_o F2$  values are higher than those in summer (this effect is believed to be really an electron depression in summer, see Wright (1962)), and a semiannual effect in  $f_o F2$ , with maxima in March and October. The anomaly is an annual effect and its relation to an annual change in geomagnetic activity cannot be studied by using the Ap index. However, the semiannual effect in  $f_o F2$  is probably closely related to the semiannual variation of Ap and other measures of the geomagnetic field. Apart from the annual and seasonal effects, there has been little detailed investigation of periodic changes in the ionosphere. In this respect the field of ionospheric studies lags behind that of geomagnetic fluctuations and there is some scope for interaction. As was mentioned in the introduction, there are likely to be ionospheric counterparts for each well-defined geomagnetic period. A study of the

phase differences for a given period could give information relevant to ionospheric dynamics. The type of spectrum analysis used for this report is well suited for ionosonde data. For example, daily noon-time values of  $f_o F2$  could be analyzed for periods varying from a few days to several months. Monthly average data, if of sufficient length, could give information about the periods greater than two months. Some ionosondes have an output every quarter-hour and the analysis of these data would cover periods as short as one half hour, something not generally possible with the standard geomagnetic indices.

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